INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Second Year, Second Semester, 2021-22
Statistics - II, Final Examination, May 11, 2022
Answer all questions   Maximum Marks: 50   Time: 3 hours

1. Let $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ be independent random samples, respectively, from $N(\mu, 1)$ and $N(2\mu, 1)$, where $-\infty < \mu < \infty$ is the unknown parameter.
(a) Derive the most powerful (MP) test for testing $H_0 : \mu = 0$ versus $H_1 : \mu = 1$ at the significance level α.
(b) Is this also the uniformly most powerful (UMP) test for testing $H_0 : \mu \leq 0$ versus $H_1 : \mu > 0$ at the significance level α? Justify? [9+6]

2. Let X have the p.d.f.
$f(x|\theta) = \theta x^{\theta-1}$, $0 < x < 1$,
where $\theta > 0$ is the unknown parameter.
(a) Find $I(\theta)$, the Fisher information number of θ contained in X.
Consider a sequence of i.i.d observations from the distribution of X.
(b) Compute the asymptotic relative efficiency of the MLE with respect to the method of moments estimator for estimating θ. [5+10]

3. $X_1, X_2, \ldots, X_n$ is a random sample from $N(\mu, \sigma^2)$ where both μ and σ^2 are unknown; $-\infty < \mu < \infty$, $\sigma^2 > 0$.
(a) Derive the generalized likelihood ratio test (GLRT) for testing $H_0 : \sigma^2 = 1$ versus $H_1 : \sigma^2 \neq 1$ at the significance level α.
(b) Construct a $100(1-\alpha)\%$ confidence interval for σ^2. [7+3]

4. Suppose that X, the number of radio-active particles emitted by a source during unit time, has the Poisson distribution with parameter λ. Assume that the prior distribution on λ is Exponential with mean 1.
(a) Find the posterior distribution of λ if $X = 1$ is observed.
(b) How does one construct the $100(1-\alpha)\%$ HPD credible interval for λ if $X = 1$ is observed? [6+4]